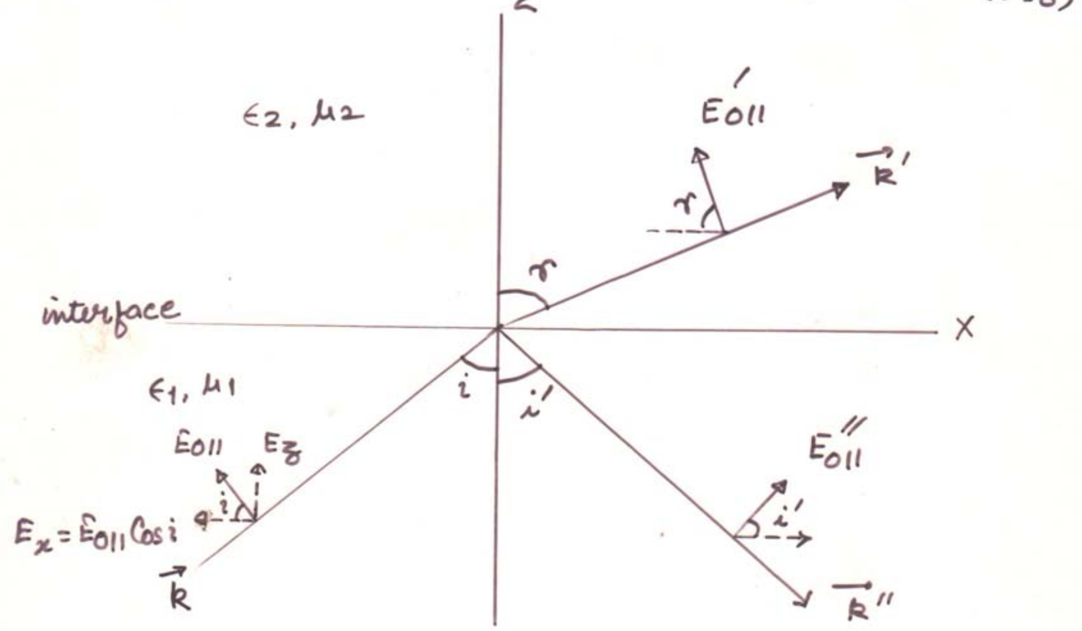
Dynamical Properties

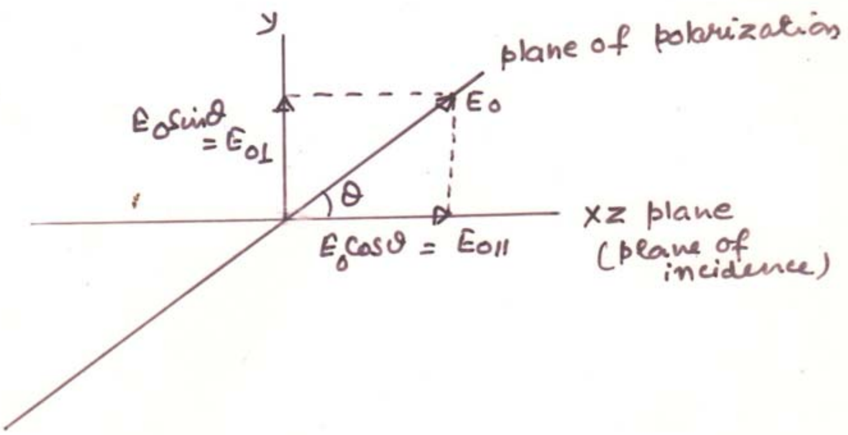
Assume the plane of incidence to be xz plane.

Boundary conditions –

(i) Tangential components of $\vec{E}$ and $\vec{H}$ vary continuously.

(ii) Normal component of $\vec{B}$ and $\vec{D}$ vary continuously. $(\sigma = 0)$





Tangential component of $\vec{E}$ (continuous)

For y component
$$E_{0\perp} + E''_{0\perp} = E'_{0\perp} \quad \text{—— (1)}$$

For x component
$$E_{0\parallel} \cos i - E''_{0\parallel} \cos i = E'_{0\parallel} \cos r \quad \text{—— (2)}$$

Similarly for H
$$H_{0\perp} + H''_{0\perp} = H'_{0\perp} \quad \text{—— (3)}$$
$$H_{0\parallel} \cos i - H''_{0\parallel} \cos i = H'_{0\parallel} \cos r \quad \text{—— (4)}$$

$$\vec{B} = \mu \vec{H} = \sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}}{k} \qquad \vec{H} = \frac{\vec{k}\times\vec{E}}{\mu\omega}$$

$$\mu\vec{H} = \cancel{\mu}\frac{\vec{k}\times\vec{E}}{\cancel{\mu}\omega} = \sqrt{\mu\epsilon}\,\frac{\vec{k}\times\vec{E}}{k} \;\Downarrow\; \vec{H} = \sqrt{\frac{\epsilon}{\mu}}\,\frac{\vec{k}\times\vec{E}}{k}$$

$$\left\{\begin{array}{l} \vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t \\ \vec{\nabla}\times\vec{H} = \partial\vec{D}/\partial t \\ e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ \partial/\partial t \rightarrow -i\omega \\ \nabla \rightarrow i\vec{k} \end{array}\right.$$

$$\vec{H} = \sqrt{\frac{\epsilon}{\mu}}\,\frac{\vec{k}\times\vec{E}}{k}$$

$$H_{0\perp} = \sqrt{\frac{\epsilon}{\mu}}\left|\frac{\vec{k}\times\vec{E}_{0\parallel}}{k}\right| \qquad \vec{k} \perp \vec{E}_{0\parallel} \qquad \vec{k}\times\vec{E}_{0\parallel} = k E_{0\parallel}$$

(Component of H_0 normal to plane of incidence)

$$\left.\begin{array}{l} H_{0\perp} = \sqrt{\frac{\epsilon}{\mu}}\, E_{0\parallel} \\ \text{and} \quad H_{0\parallel} = \sqrt{\frac{\epsilon}{\mu}}\, E_{0\perp} \end{array}\right\}$$

Eqns (3) & (4) $\Rightarrow$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\left(E_{0\parallel} + E''_{0\parallel}\right) = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E'_{0\parallel} \qquad (3a)$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\left(E_{0\perp} - E''_{0\perp}\right)\cos i = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E'_{0\perp}\cos r \qquad (4a)$$

$$\mu_1 = \mu_2$$

$$\sqrt{\epsilon_1}\, E_{0\parallel} + \sqrt{\epsilon_1}\, E''_{0\parallel} = \sqrt{\epsilon_2}\, E'_{0\parallel} \qquad (3b)$$

$$\left(\sqrt{\epsilon_1}\, E_{0\perp} - \sqrt{\epsilon_1}\, E''_{0\perp}\right)\cos i = \sqrt{\epsilon_2}\, E'_{0\perp}\cos r \qquad (4b)$$

TE Mode $\rightarrow$ Electric field component is normal to plane of incidence xz i.e. parallel to boundary plane xy

$$\sqrt{\epsilon_1}\left(E_{0\perp} - E''_{0\perp}\right)\cos i = \sqrt{\epsilon_2}\, E'_{0\perp}\cos r$$

$$\text{and} \quad E_{0\perp} + E''_{0\perp} = E'_{0\perp}$$

$$\Rightarrow \left.\begin{array}{l} \sqrt{\epsilon_2}\, E'_{0\perp}\cos r + \sqrt{\epsilon_1}\, E''_{0\perp}\cos i = \sqrt{\epsilon_1}\, E_{0\perp}\cos i \\ E'_{0\perp} - E''_{0\perp} = E_{0\perp} \end{array}\right\}$$

$$\Rightarrow \quad \frac{E'_{0\perp}}{E_{0\perp}} = \frac{2\sqrt{\epsilon_1}\cos i}{\sqrt{\epsilon_1}\cos i + \sqrt{\epsilon_2}\cos r}$$

$$\left\{\begin{array}{l} \dfrac{E'_{0\perp}}{E_{0\perp}} = \dfrac{2 n_1 \cos i}{n_1\cos i + n_2 \cos r} \\[2ex] \dfrac{E''_{0\perp}}{E_{0\perp}} = \dfrac{n_1\cos i - n_2\cos r}{n_1\cos i + n_2\cos r} \end{array}\right\} \quad (\mathrm{I})$$

TM Mode (component of $\vec{E}$, in the plane of incidence)

$$(E_{011} - E_{011}'') \cos i = E_{011}' \cos r$$

$$\sqrt{\epsilon_1} (E_{011} + E_{011}'') = \sqrt{\epsilon_2} E_{011}'$$

Rearranging.

$$E_{011}' \cos r + E_{011}'' \cos i = E_{011} \cos i$$

$$\sqrt{\epsilon_2} E_{011}' - \sqrt{\epsilon_1} E_{011}'' = \sqrt{\epsilon_1} E_{011}$$

$$\Rightarrow \quad \frac{E_{011}'}{E_{011}} = \frac{2\sqrt{\epsilon_1} \cos i}{\sqrt{\epsilon_1} \cos r + \sqrt{\epsilon_2} \cos i}$$

$$\frac{E_{011}''}{E_{011}} = \frac{\sqrt{\epsilon_2} \cos i - \sqrt{\epsilon_1} \cos r}{\sqrt{\epsilon_2} \cos i + \sqrt{\epsilon_1} \cos r}$$

$$\Rightarrow \quad \left.\begin{aligned} \frac{E_{011}'}{E_{011}} &= \frac{2n_1 \cos i}{n_1 \cos r + n_2 \cos i} \\ \frac{E_{011}''}{E_{011}} &= \frac{n_2 \cos i - n_1 \cos r}{n_2 \cos i + n_1 \cos r} \end{aligned}\right\} \quad \text{II}$$

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \Rightarrow \begin{array}{l}\sin i \propto n_2 \\ \sin r \propto n_1\end{array}$$

We thus have

FRESNEL'S AMPLITUDE RELATIONS

No phase change on refraction

$$\frac{E'_{0\perp}}{E_{0\perp}} = \frac{2\sin r\cos i}{\sin r\cos i + \sin i\cos r} = \frac{2\sin r\cos i}{\sin(i+r)}$$

$$\frac{E''_{0\perp}}{E_{0\perp}} = \frac{\sin r\cos i - \sin i\cos r}{\sin r\cos i + \sin i\cos r} = -\frac{\sin(i-r)}{\sin(i+r)}$$

phase change of π

(If $i > r$, $n_2 > n_1$, rarer to denser)

$$\frac{E'_{0\parallel}}{E_{0\parallel}} = \frac{2\sin r\cos i}{\sin i\cos i + \cos r\sin r} = \frac{2\sin r\cos i}{\sin(i+r)\cos(i-r)}$$

no phase change

$$\boxed{\frac{E''_{0\parallel}}{E_{0\parallel}} = \frac{\sin i\cos i - \sin r\cos r}{\sin i\cos i + \sin r\cos r} = \frac{\tan(i-r)}{\tan(i+r)}}$$

$\Rightarrow i + r = \pi/2$

i.e. $i = \frac{\pi}{2} - r = i_p$

Then, $E''_{0\parallel} = 0$

Polarized Reflected Wave.

$i_p \equiv$ Brewster angle

Polarization by reflection

Reflected wave is plane-polarized with polarization vector perpendicular to plane of incidence.

At normal incidence, $i = 0 = i' = r$

Set (I) and (II) $\Longrightarrow$

$$\frac{E'_{0\perp}}{E_{0\perp}} = \frac{2n_1}{n_1+n_2} = \frac{E'_{0\parallel}}{E_{0\parallel}}$$

$$\frac{E''_{0\parallel}}{E_{0\parallel}} = \frac{n_2-n_1}{n_2+n_1} = -\frac{E''_{0\perp}}{E_{0\perp}}$$

* ${}_1n_2 = \frac{n_2}{n_1} = \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\cos i_p} = \tan i_p$

At Grazing angle of incidence

$i = \pi/2$ and therefore $\frac{E'_{0\perp}}{E_{0\perp}} = 0$ & $\frac{E'_{0\parallel}}{E_{0\parallel}} = 0$

Set (I) and (II) $\Longrightarrow$

$\Rightarrow$ Total internal reflection regardless of parallel or perpendicular polarization.

* Air to glass $i_p = \tan^{-1}(1.5) \approx 57^\circ$

Glass to air $i_p = \tan^{-1}(1/1.5) \approx 33^\circ$